# Numbering System Binary , Decimal , Octal , Hexadecimal

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# The decimal system (base 10)

- The word decimal is derived from the Latin root decem (ten). In this system the base b = 10 and we use ten symbols

S = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

- The symbols in this system are often referred to as decimal digits or just digits.

# The binary system (base 2)

- The word binary is derived from the Latin root bini (or two by two). In this system the base b = 2 and we use only two symbols,

$$S = \{0, 1\}$$

- The symbols in this system are often referred to as binary digits or bits (binary digit).

# The hexadecimal system (base 16)

- The word hexadecimal is derived from the Greek root hex (six) and the Latin root decem (ten). In this system the base b = 16 and we use sixteen symbols to represent a number. The set of symbols is

S = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F}

- Note that the symbols A, B, C, D, E, F are equivalent to 10, 11, 12, 13, 14, and 15 respectively. The symbols in this system are often referred to as hexadecimal digits.

# The octal system (base 8)

- The word octal is derived from the Latin root octo (eight). In this system the base $b = 8$ and we use eight symbols to represent a number. The set of symbols is

$$S = \{0, 1, 2, 3, 4, 5, 6, 7\}$$

# Summary of the four positional systems

- Table 2.1 shows a summary of the four positional number systems discussed before.

**Table 2.1** Summary of the four positional number systems

| *System* | *Base* | *Symbols* | *Examples* |
|---|---|---|---|
| Decimal | 10 | 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 | 2345.56 |
| Binary | 2 | 0, 1 | $(1001.11)_2$ |
| Octal | 8 | 0, 1, 2, 3, 4, 5, 6, 7 | $(156.23)_8$ |
| Hexadecimal | 16 | 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F | $(A2C.A1)_{16}$ |

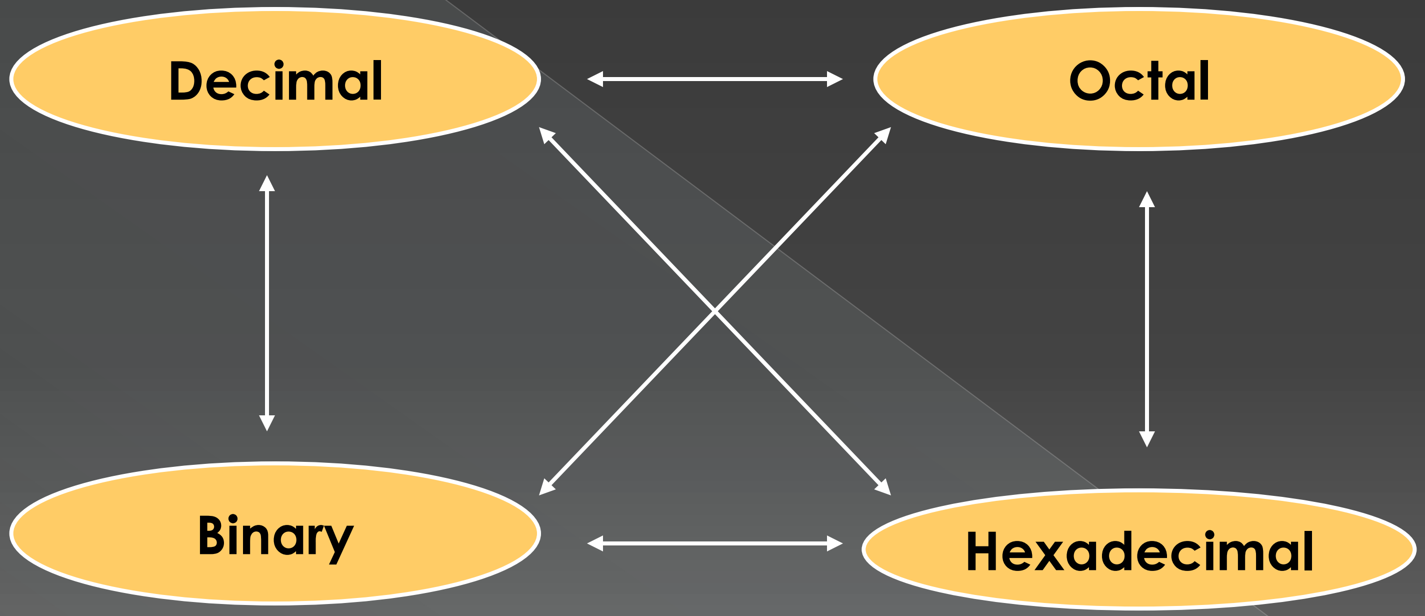
- Table 2.2 shows how the number 0 to 15 is represented in different systems.

**Table 2.2** Comparison of numbers in the four systems

| *Decimal* | *Binary* | *Octal* | *Hexadecimal* |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 |
| 2 | 10 | 2 | 2 |
| 3 | 11 | 3 | 3 |
| 4 | 100 | 4 | 4 |
| 5 | 101 | 5 | 5 |
| 6 | 110 | 6 | 6 |
| 7 | 111 | 7 | 7 |
| 8 | 1000 | 10 | 8 |
| 9 | 1001 | 11 | 9 |
| 10 | 1010 | 12 | A |
| 11 | 1011 | 13 | B |
| 12 | 1100 | 14 | C |
| 13 | 1101 | 15 | D |
| 14 | 1110 | 16 | E |
| 15 | 1111 | 17 | F |

# Conversion Among Bases

- The possibilities:

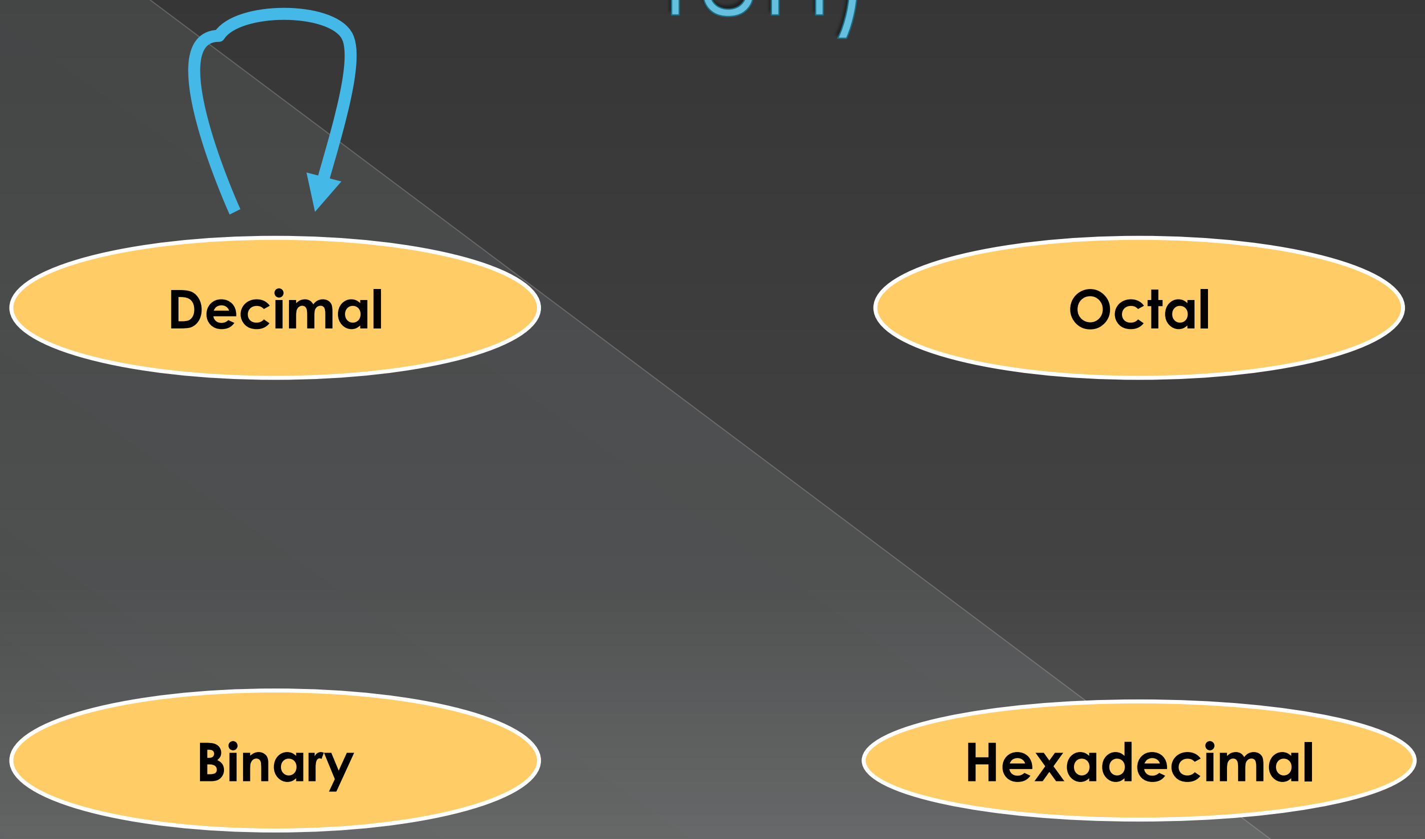


# Quick Example

$$25_{10} = 11001_2 = 31_8 = 19_{16}$$

Base

# Decimal to Decimal (just for fun)

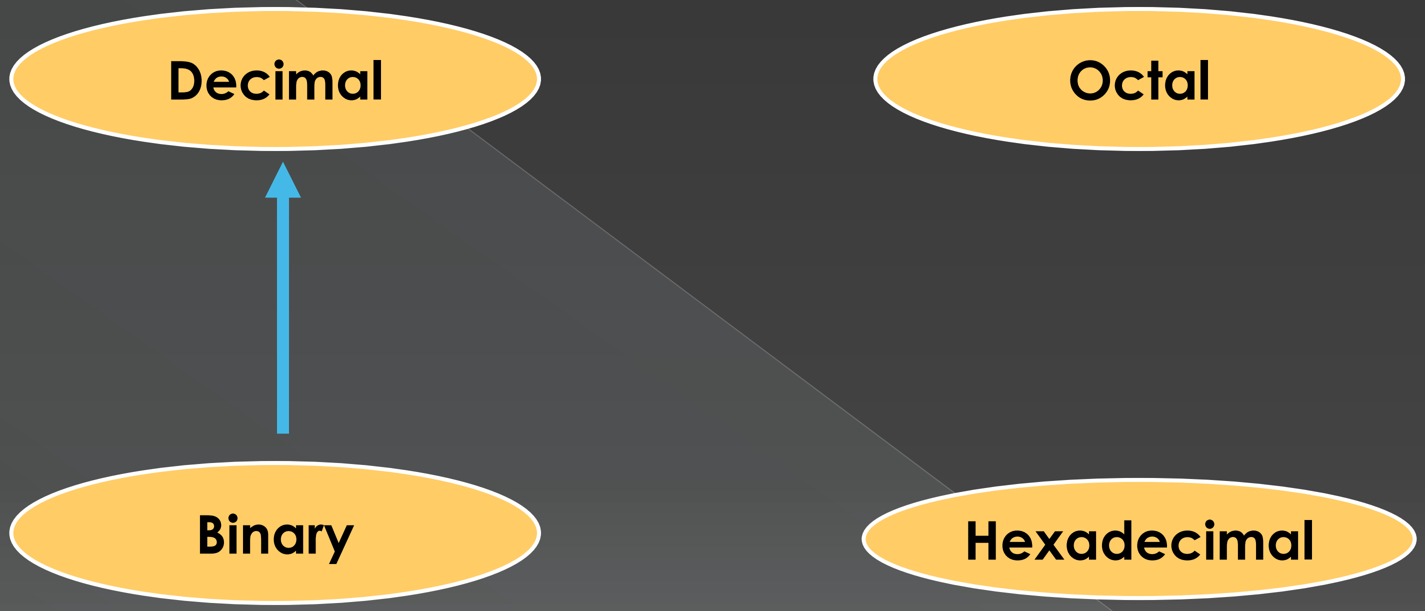


$$125_{10} \Rightarrow \begin{array}{l} 5 \times 10^0 = 5 \\ 2 \times 10^1 = 20 \\ 1 \times 10^2 = 100 \\ \hline \quad\quad\quad\quad\quad 125 \end{array}$$

Weight

Base

# Binary to Decimal



# Binary to Decimal

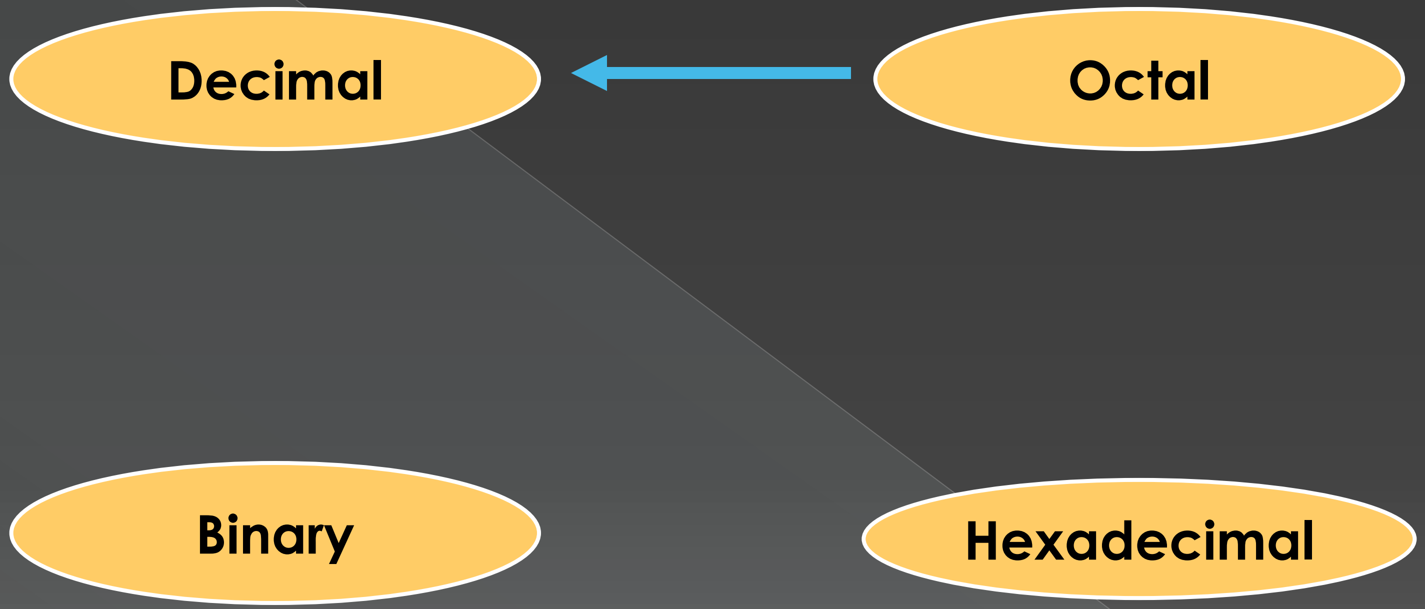
- Technique
  - Multiply each bit by $2^n$, where *n* is the "weight" of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

# Example 1:

Bit "0"

$$101011_2 \Rightarrow \begin{array}{l} 1 \times 2^0 = 1 \\ 1 \times 2^1 = 2 \\ 0 \times 2^2 = 0 \\ 1 \times 2^3 = 8 \\ 0 \times 2^4 = 0 \\ 1 \times 2^5 = 32 \\ \hline 43_{10} \end{array}$$

# Octal to Decimal



# Octal to Decimal

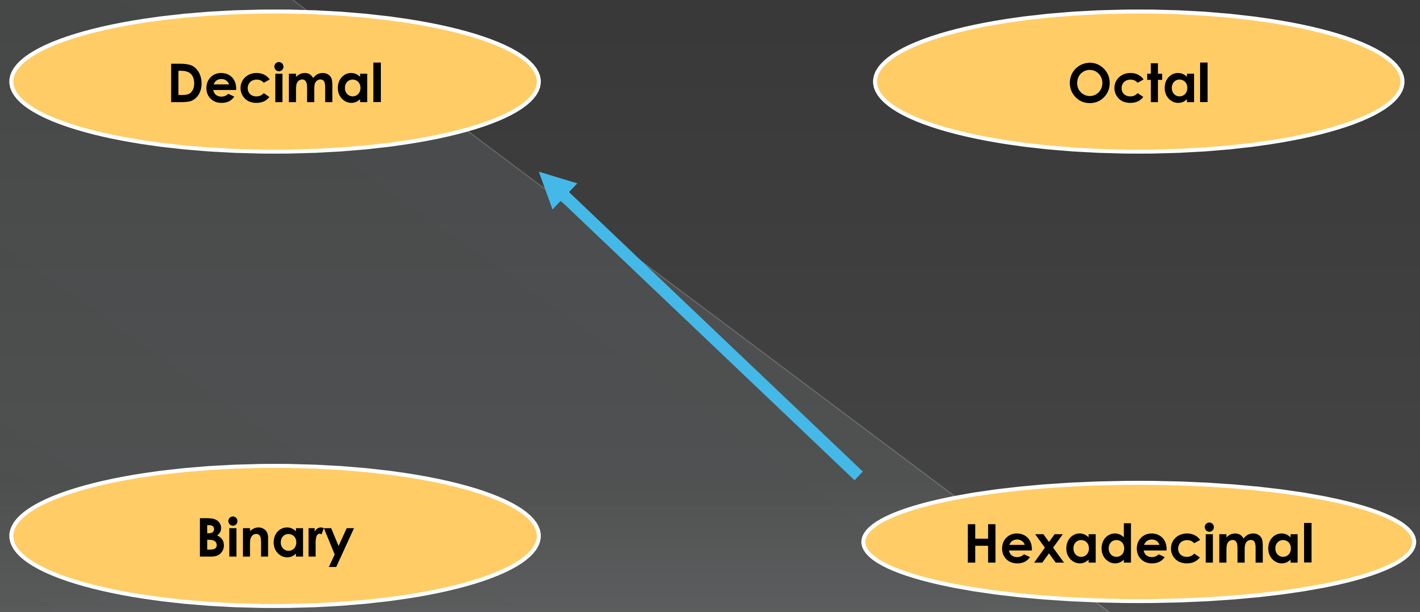
- Technique
  - Multiply each bit by $8^n$, where $n$ is the "weight" of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

# Example 2:

$724_8 \Rightarrow$

| | |
|---|---|
| $4 \times 8^0 =$ | 4 |
| $2 \times 8^1 =$ | 16 |
| $7 \times 8^2 =$ | 448 |
| | $468_{10}$ |

# Hexadecimal to Decimal



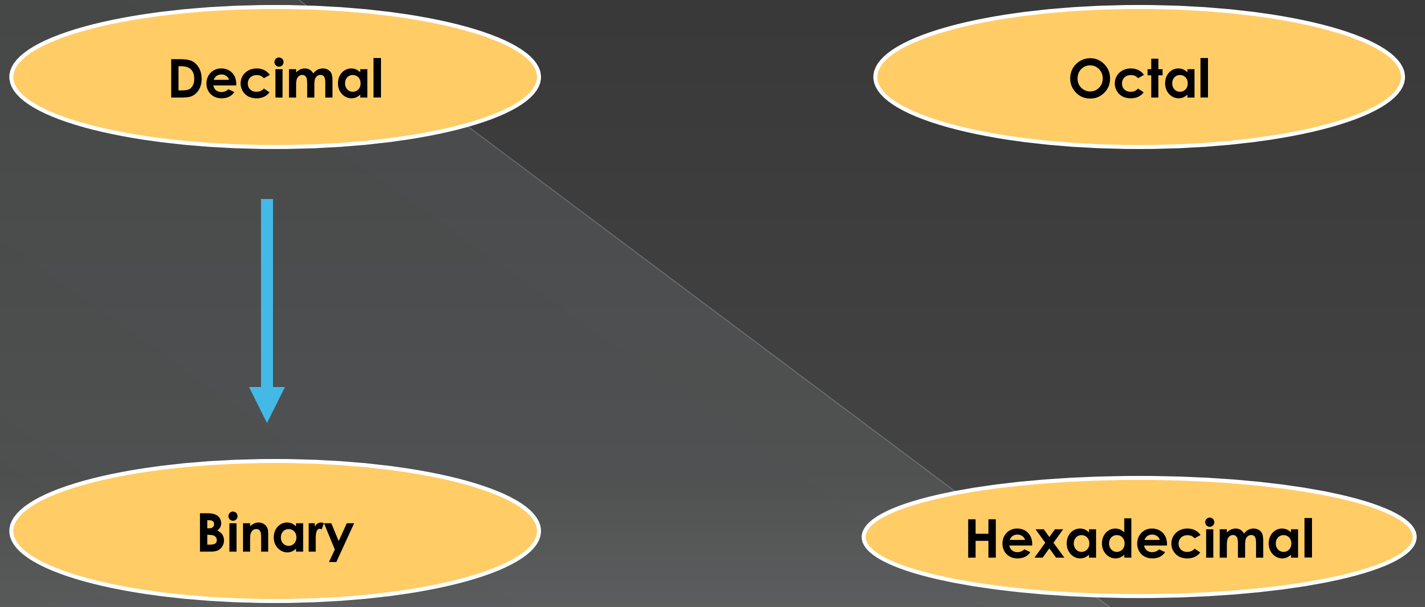
# Hexadecimal to Decimal

- Technique
  - Multiply each bit by $16^n$, where $n$ is the "weight" of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

# Example 3:

$$
\begin{aligned}
ABC_{16} \Rightarrow \quad & C \times 16^0 = 12 \times 1 = 12 \\
& B \times 16^1 = 11 \times 16 = 176 \\
& A \times 16^2 = 10 \times 256 = 2560 \\
\hline
& 2748_{10}
\end{aligned}
$$

# Decimal to Binary



# Decimal to Binary

- Technique
  - Divide by two, keep track of the remainder
  - First remainder is bit 0 (LSB, least-significant bit)
  - Second remainder is bit 1
  - Etc.

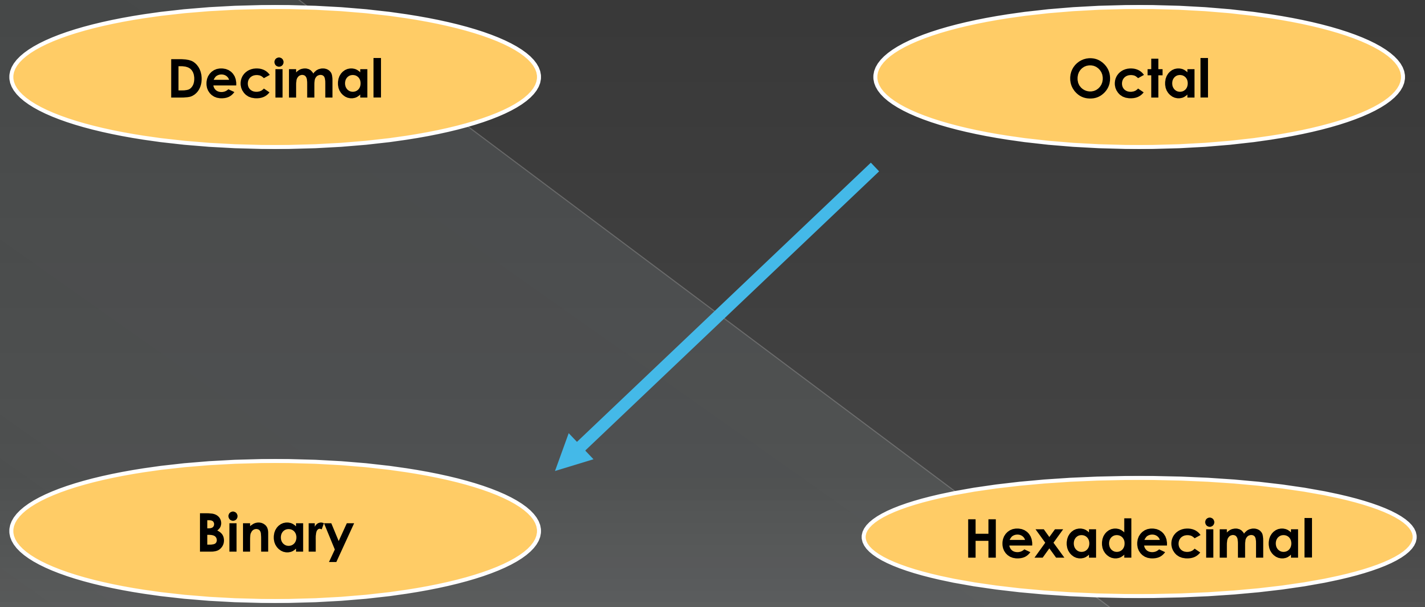
# Example 4:

$125_{10} = ?_2$

| | | |
|---|---|---|
| 2 | 125 | |
| 2 | 62 | 1 |
| 2 | 31 | 0 |
| 2 | 15 | 1 |
| 2 | 7 | 1 |
| 2 | 3 | 1 |
| 2 | 1 | 1 |
| | 0 | 1 |

$125_{10} = 1111101_2$

# Octal to Binary



# Octal to Binary

- Technique
  - Convert each octal digit to a 3-bit equivalent binary representation

# Example 5:

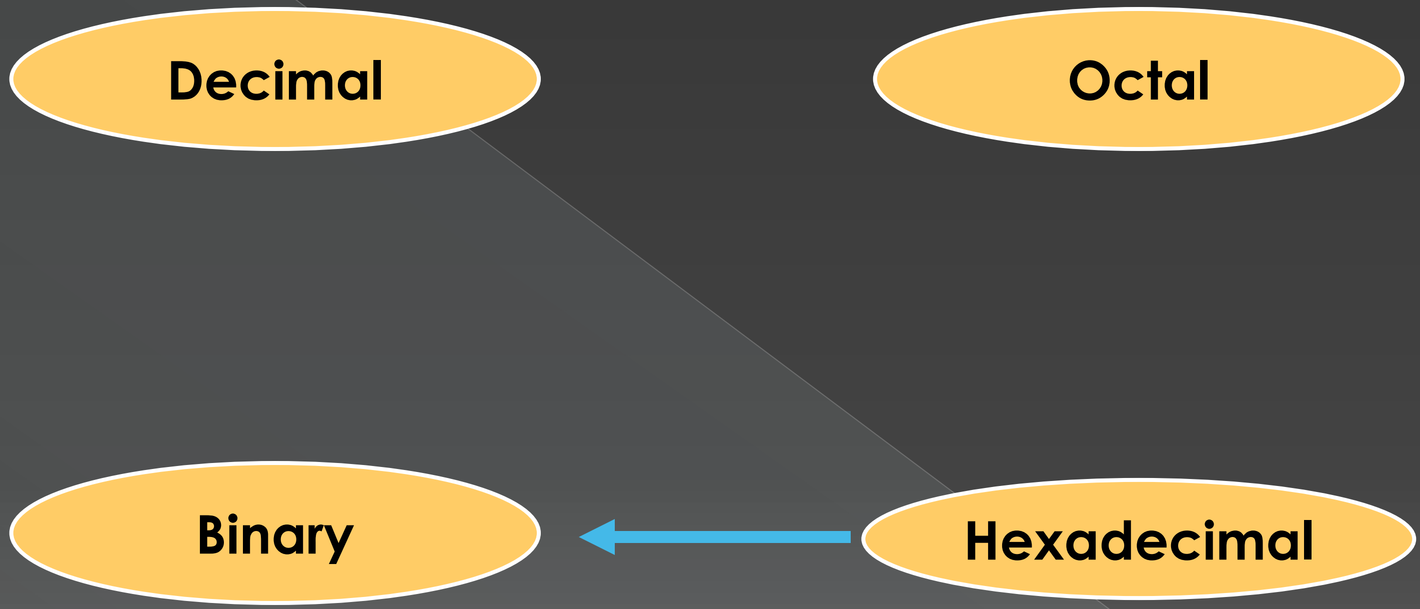
$705_8 = ?_2$

7 → 111
0 → 000
5 → 101

111 000 101

$705_8 = 111000101_2$

# Hexadecimal to Binary



# Hexadecimal to Binary

- Technique
  - Convert each hexadecimal digit to a 4-bit equivalent binary representation

# Example 6:

$10AF_{16} = ?_2$

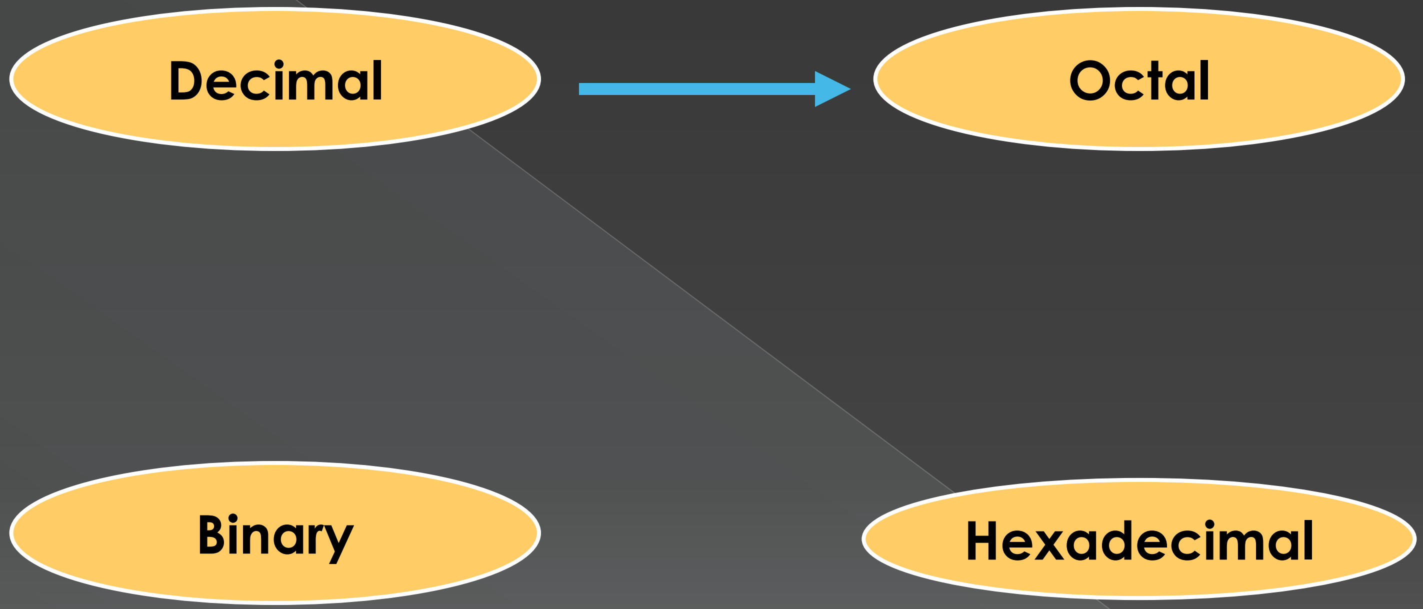
1 0 A F

↓ ↓ ↓ ↓

0001 0000 1010 1111

$10AF_{16} = 0001000010101111_2$

# Decimal to Octal



# Decimal to Octal

- Technique
  - Divide by 8
  - Keep track of the remainder

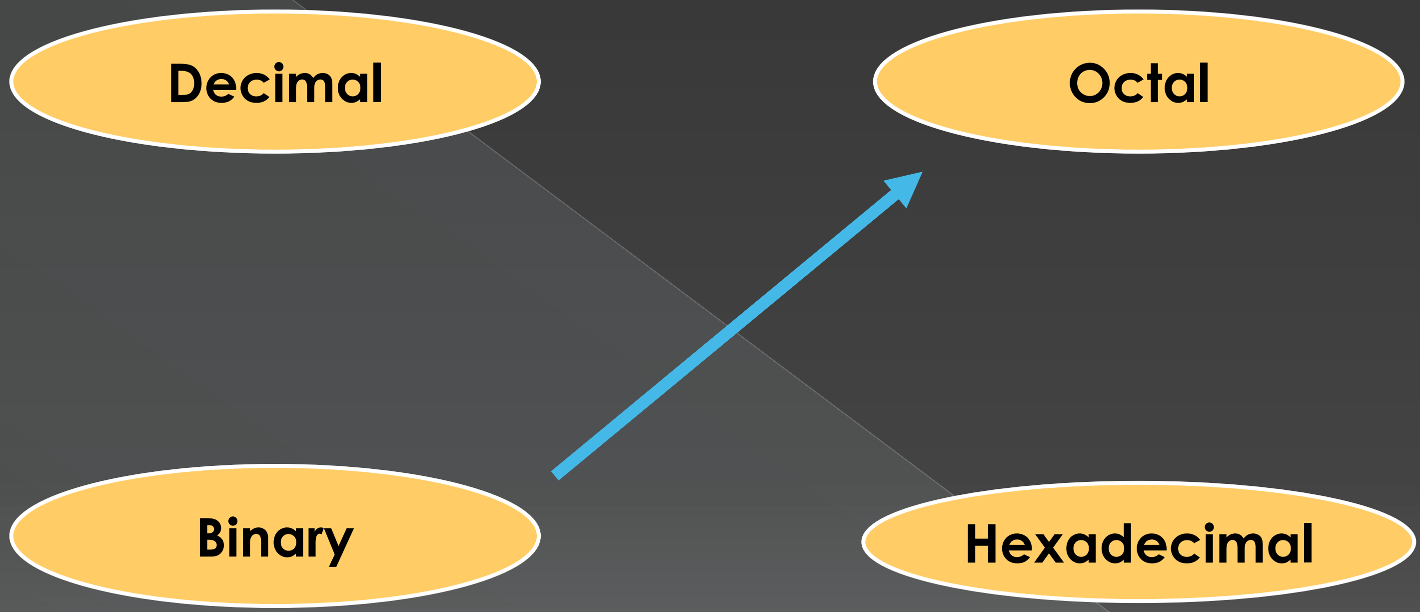
# Example 7:

$1234_{10} = ?_8$

| | | |
|---|---|---|
| 8 | 1234 | |
| 8 | 154 | 2 |
| 8 | 19 | 2 |
| 8 | 2 | 3 |
| | 0 | 2 |

$1234_{10} = 2322_8$

# Binary to Octal



# Binary to Octal

- Technique
  - Group bits in threes, starting on right
  - Convert to octal digits

# Example 8:

$1011010111_2 = ?_8$

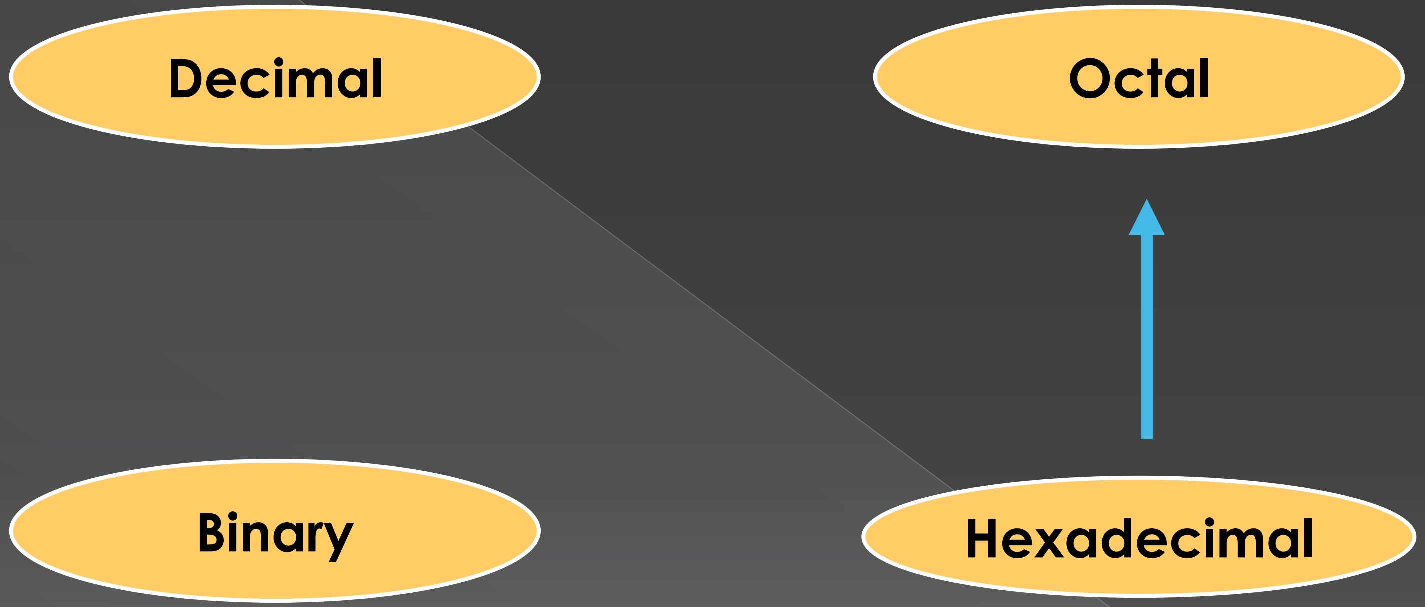
1 011 010 111

↓ ↓ ↓ ↓

1 3 2 7

$1011010111_2 = 1327_8$

# Hexadecimal to Octal



# Hexadecimal to Octal

- Technique
  - Use binary as an intermediary

# Example 9:

$1F0C_{16} = ?_8$

1 F 0 C

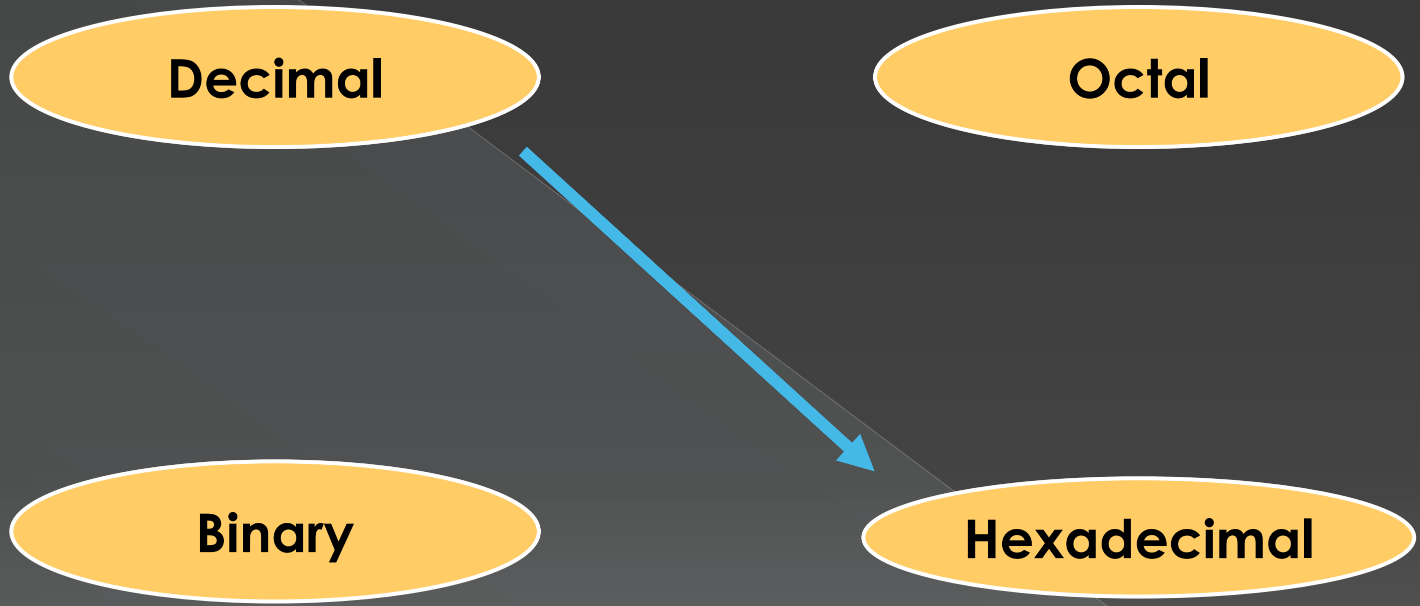
0001 1111 0000 1100

0 | 001 | 111 | 100 | 001 | 100

1 7 4 1 4

$1F0C_{16} = 17414_8$

# Decimal to Hexadecimal



# Decimal to Hexadecimal

- Technique
  - Divide by 16
  - Keep track of the remainder

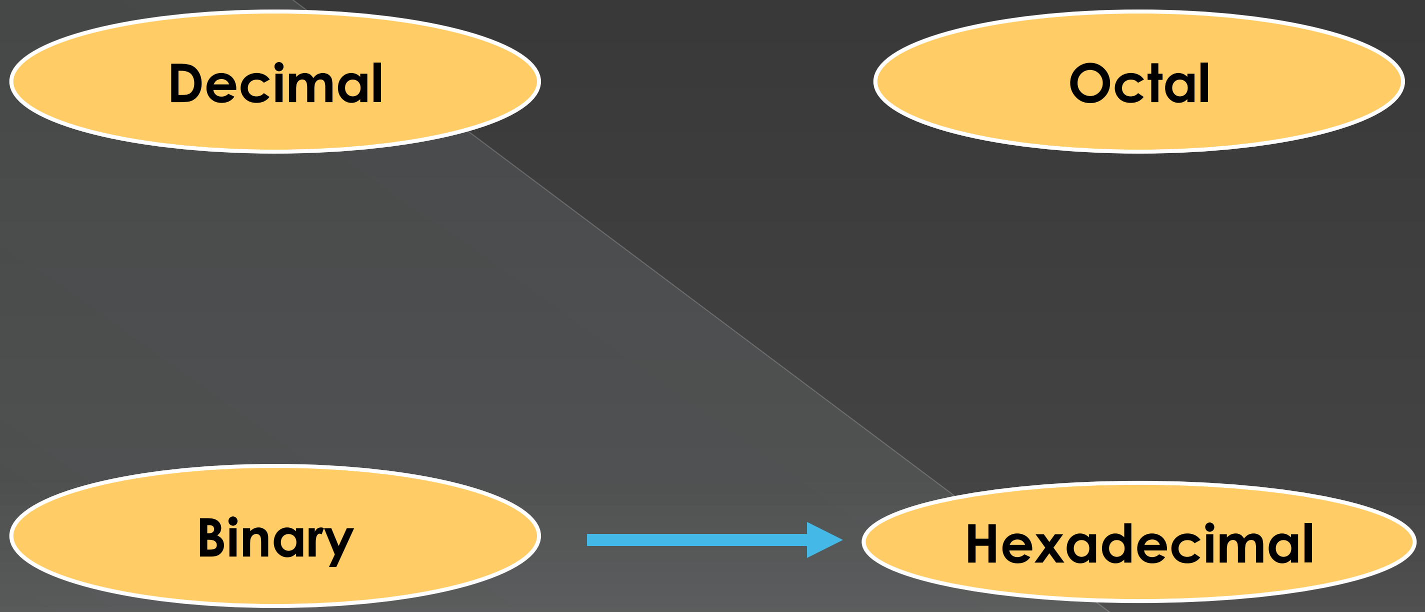
# Example 10:

$1234_{10} = ?_{16}$

| | | |
|---|---|---|
| 16 | 1234 | |
| 16 | 77 | 2 |
| 16 | 4 | 13 = D |
| | 0 | 4 |

$1234_{10} = 4D2_{16}$

# Binary to Hexadecimal



# Binary to Hexadecimal

- Technique
  - Group bits in fours, starting on right
  - Convert to hexadecimal digits

# Example 11:

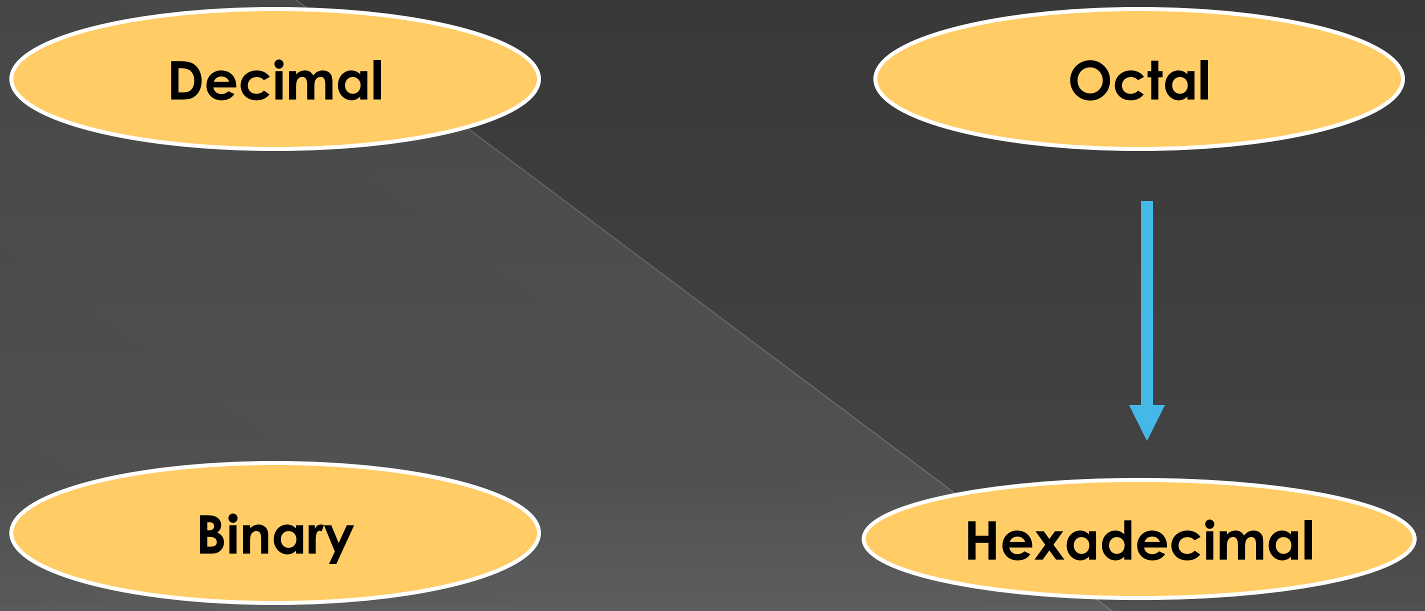
$1010111011_2 = ?_{16}$

10 1011 1011

2 B B

$1010111011_2 = 2BB_{16}$

# Octal to Hexadecimal



# Octal to Hexadecimal

- Technique
  - Use binary as an intermediary

# Example 12:

$1076_8 = ?_{16}$

1 0 7 6

001 000 111 110

2 3 E

$1076_8 = 23E_{16}$

# Exercise – Convert ...

| Decimal | Binary | Octal | Hexa-decimal |
| --- | --- | --- | --- |
| 33 | | | |
| | 1110101 | | |
| | | 703 | |
| | | | 1AF |

Don't use a calculator!

# Exercise – Convert …

Answer

| Decimal | Binary | Octal | Hexa-decimal |
|---|---|---|---|
| 33 | 100001 | 41 | 21 |
| 117 | 1110101 | 165 | 75 |
| 451 | 111000011 | 703 | 1C3 |
| 431 | 110101111 | 657 | 1AF |

# Thanks